Composite Structures 92 (2010) 1269–1281

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Free vibration response of composite sandwich cylindrical shell with flexible core

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ARTICLE INFO

Article history:
Available online 18 November 2009

Keywords:
Polymer matrix composite
Flexible core
Foam
Free vibration
Cylindrical sandwich shell
Analytical analysis

ABSTRACT

This study deals with the free vibration analysis of composite sandwich cylindrical shell with a flexible core using a higher order sandwich panel theory. The formulation uses the classical shell theory for the face sheets and an elasticity theory for the core and includes derivation of the governing equations along with the appropriate boundary conditions. The model consists of a systematic approach for the analysis of sandwich shells with a flexible core, having high-order effects caused by the nonlinearity of the in-plane and the vertical displacements of the core. The behavior is presented in terms of internal resultants and displacements in the faces, peeling and shear stresses in the face–core interface and stress and displacement field in the core. The accuracy of the solution is examined by comparing the results obtained with the analytical and numerical results published in the literatures. The parametric study is also included to investigate the effect of geometrical properties such as radius of curvature, length and sector angle of the shell.

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1. Introduction

Sandwich structures with laminated polymer matrix composite face sheets and a foam or low strength honeycomb core are being used increasingly in aerospace, automobile, locomotive and construction industries for their excellent properties. This type of core is flexible as compared to face sheets. This behavior of the core in such structures is accounted for in the vertical direction and it significantly affects the overall behavior of the structures under various loading schemes. This may lead to different behavior patterns in the outer and the inner face sheets as compared with the panels whose core is considerably stiff in the out-of-plane direction.

The classical approach to the response of sandwich structures resorts to decoupling of the local and the global responses, whence ignoring the interaction between them. At present, the analytical models of the structures are largely based on one of the following approaches: classical shell theory [1], elastic foundation model [2], various equivalent single layer and shear deformation theories [3–6], and most recently, the high-order sandwich panel theory (HSAPT) [7,8].

Qatu in his review article [9] and his recent published book [10], surveyed the literatures on the dynamic behavior of laminated shells. The review has been conducted with emphasis given to the theory being applied (thin, thick, 3D, nonlinear), the analysis method (exact, Ritz, finite elements), complicating effects (initial stress, imperfection, added masses and springs, elastic supports, rotating shells, and others) and the various shell geometries that are related to vibration studies (cylindrical, conical, spherical, and others).

Khare et al. [3] used the higher order shear deformation theories for thermo-mechanical and free vibration analysis of laminated sandwich thick shells. Garg et al. [4] investigated the free vibration analysis of simply supported composite and sandwich doubly curved shells. Their formulation included the Sander's theory and they assumed a parabolic distribution of transverse shear strains through the shell thickness. Singh [5] studied the free vibration of open deep sandwich shells made of thin layers and a moderately thick core. He used Rayleigh–Ritz method to obtain the natural frequencies. Korjakin et al. [6] used a zig–zag model to investigate the free damped vibrations of sandwich shells. They performed the vibration analysis with damping consideration of cylindrical, conical and spherical sandwich shells.

Frostig et al. [7,8] developed a high-order theory for sandwich panels. The theory does not impose any restriction on the distribution of the deformation through the thickness and the high-order effects are the results of the theory and not a prior assumption for displacement fields. Khalili et al. [11] and Malekzadeh et al.

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doi:10.1016/j.compstruct.2009.10.021

Nomenclature

x, ϕ, z	longitudinal, circumferential and radial coordinates
$i(=c, t, b)$	indices for core, outer (top) and inner (bottom) face sheets
u_i, v_i, w_i	displacements in longitudinal, circumferential and radial directions
u_{0i}, v_{0i}	mid-plane displacements in longitudinal and circumferential directions
ε_{xxi}, $\varepsilon_{\phi\phi i}$, $\gamma_{x\phi i}$	strains in face sheets
ε_{xx0i}, $\varepsilon_{\phi\phi 0i}$, $\gamma_{x\phi 0i}$	mid-plane strains in face sheets
κ_{xxi}, $\kappa_{\phi\phi i}$, $\kappa_{x\phi i}$	mid-plane curvatures in face sheets
β_{xxi}, $\beta_{\phi\phi i}$	rotations of the face sheets
σ_{xxi}, $\sigma_{\phi\phi i}$, $\tau_{x\phi i}$	in-plane normal and shear stresses in the face sheets
ε_{xxi}, $\varepsilon_{\phi\phi i}$, $\gamma_{x\phi i}$	in-plane normal and shear strains in the face sheets
σ_{rrc}, τ_{xrc}, $\tau_{\phi rc}$	radial and shear stresses in the core
ε_{rrc}, γ_{xrc}, $\gamma_{\phi rc}$	radial and shear strains in the core
E_c, $G_{\phi c}$, G_{xc}	Young's and shear modulus for the core
N_x, N_ϕ, $N_{x\phi}$	stress resultants
M_x, M_ϕ, $M_{x\phi}$	moment resultants
A_{ij}	extensional stiffness matrix
B_{ij}	bending–stretching coupling matrix
D_{ij}	flexural stiffness matrix
δ	variational operator
T	kinetic energy
U	potential energy
V_i	volumes of the face sheets and the core
r_i	radii of curvature for each face sheet
d_i	thickness of the outer and the inner face sheets
r_{ic}	radii of curvature at core–face interfaces
h_c, h_f	thickness of the core and the face sheets
h	total thickness of sandwich shell
L	length of shell
a	width of shell
α_0	sector angle of shell
I_{ji}	moment of inertia
ρ_i	mass density
K_i, k_0, c_i	constants
$\tilde{M}$	mass matrix
$\tilde{K}$	stiffness matrix
j	imaginary unit
ω	natural frequency
$\bar{\omega}$	dimensionless natural frequency
m,n	wave numbers

[12] modified HSAPT by applying first-order shear deformation theory for face sheets and used improved HSPAT to study the free vibration and low velocity response of sandwich panels. Frostig and Baruch [13] used HSAPT to analyse the free vibrations of straight sandwich beams with a transversely flexible core. Rahmani et al. [14] applying HSAPT studied vibration behavior of sandwich structure with a flexible functionally graded syntactic core. Bozhevolnaya and Frostig [15] presented a high-order model for free vibration of single curved sandwich beam. Frostig and Thomsen [16] studied free vibration of sandwich panels with a flexible core based on HSAPT. They considered two types of computational models. The first model used the vertical shear stresses in the core, in addition to the displacements of outer and inner face sheets as its unknowns. The second model assumed a polynomial description of the displacement fields in the core that was based on the displacement fields of the first model.

Although the dynamic behavior of sandwich beams and plates with flexible core has been the subject of some studies [7,8] and [11–16], there are few reports on the free vibration of the soft core sandwich shells.

The response of the sandwich shells with soft cores should be simulated with the aid of an enhanced theory to account its radial flexibility. This radial flexibility affects the stress and displacement fields in the face sheets and leads to nonlinear in-plane and vertical displacement patterns in the core. Equivalent single layer theories do not consider the effects of transverse flexibility as well as the interaction between the face sheets and the flexible core. Most of these theories are only concerned with the analysis of the lowest vibration modes. The free vibration modes of the sandwich shells with flexible core consist of the overall and the localized modes, which the classical plate and the sandwich panel theories can not detect them.

In the present article, the formulation that includes the derivation of the governing equations of composite sandwich cylindrical shell with flexible core is presented based on high-order sandwich panel theory [7,8].

In the proposed model no prior assumptions are made on the displacement fields in the core. The displacements and the stress fields of the core are determined through the solution of its elasticity field equations along with the kinematic and constitutive relations. The nonpolynomial displacement distributions through the shell thickness are obtained as an inherent part of the solution. This nonlinear displacement distribution in the present analysis allows an accurate implementation of the effects of the flexibility of the core.

The results obtained by the present analysis are validated by the results published in the previous literatures. The effect of some parameters such as thickness ratio, radius of curvature, length and sector angle of the shell on free vibration of the composite sandwich cylindrical shell is investigated.

2. Theoretical formulation

2.1. Basic assumptions

The cylindrical sandwich shell studied in this study is composed of two FRP composite laminated face sheets and a flexible core. The panel is assumed to have a length L and a total thickness h as shown in Fig. 1, where the coordinates are also shown in the same figure. In the following, indices t and b refers to the outer (top) and the inner (bottom) face sheets of the shell, respectively. The assumptions used in the present analysis follow in general, those encountered in linear elastic small deformation theories. The face sheets are considered as ordinary thin shells with flexural and in-plane rigidities. The core behavior follows the assumption adopted by many researchers for the flexible core [7,8]. It has shear resistance, but is free of in-plane normal and shear stresses. This assumption is practically correct for foam cores, since its elastic modulus and flexural rigidity are much smaller than those of the face sheets. The core is assumed to behave in a linear elastic manner with small deformations, while its height may change and its transverse plane takes a nonlinear shape after deformation. Note that there are no prior assumptions on the deformation fields through the thickness of the core. The outer and the inner face sheets and the core are assumed to be perfect bonded: i.e. there is no relative displacement between the core and the adjacent face sheets at the interfaces.

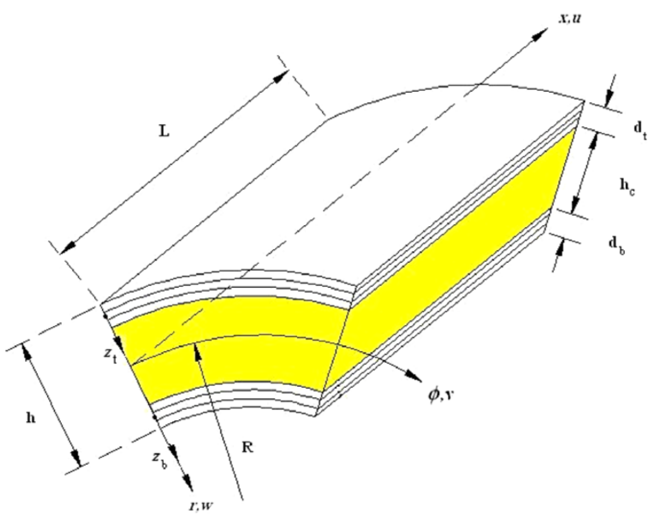

Fig. 1. Geometry and coordinates of the composite sandwich cylindrical shell.

2.2. Shell kinematics

The displacements u, v and w of the face sheets in the x (longitudinal), ϕ (circumferential) and z (radial) directions are expressed through the following relations [17]:

$$\begin{aligned} u_i(x,\phi,z,t) &= u_{0i}(x,\phi,t) + z_i\beta_{xxi} \\ v_i(x,\phi,z,t) &= v_{0i}(x,\phi,t) + z_i\beta_{\phi\phi i} \quad (i=t,b) \\ w_i(x,\phi,z,t) &= w_i(x,\phi,t) \end{aligned} \tag{1}$$

and the kinematic equations for the strains in the face sheets are as follow:

$$\varepsilon_{\phi\phi i} = \varepsilon_{\phi\phi 0i} + z_i\kappa_{\phi\phi i}, \quad \varepsilon_{xxi} = \varepsilon_{xx0i} + z_i\kappa_{xxi}, \quad \gamma_{x\phi i} = \gamma_{x\phi 0i} + z_i\kappa_{x\phi i} \tag{2}$$

where the mid-plane in-plane strains and the curvatures are

$$\begin{aligned} &\varepsilon_{xx0i} = u_{0i,x}, \quad \kappa_{xxi} = \beta_{xxi,x} \\ &\varepsilon_{\phi\phi 0i} = (v_{0i,\phi} + w_i)/r_i, \quad \kappa_{\phi\phi i} = \beta_{\phi\phi i,\phi}/r_i \\ &\gamma_{x\phi 0i} = v_{0i,x} + (u_{0i,\phi}/r_i), \quad \kappa_{x\phi i} = \beta_{\phi\phi i,x} + (\beta_{xxi,\phi}/r_i) \end{aligned} \tag{3}$$

where r_i is the radii of curvature for each face sheet and

$$\beta_{xxi} = -w_{i,x}, \quad \beta_{\phi\phi i} = (v_{0i} - w_{i,\phi})/r_i \tag{4}$$

The kinematic relations used for the core, assuming small linear deformation, take the following form:

$$\varepsilon_{rrc} = w_{c,r}, \quad \gamma_{\phi rc} = v_{c,r} + \frac{w_{c,\phi} - v_c}{r}, \quad \gamma_{xrc} = w_{c,x} + u_{c,r} \tag{5}$$

The compatibility conditions, which interconnect the face sheets and the core at the interface layers for both outer and inner skins, are:

$$\begin{aligned} u_c(r=r_{ic}) &= u_{0i} + (-1)^k(d_i/2)\beta_{xxi} \\ v_c(r=r_{ic}) &= v_{0i} + (-1)^k(d_i/2)\beta_{\phi\phi i} \\ w_c(r=r_{ic}) &= w_i \end{aligned} \tag{6}$$

where d_i (i = t, b) are respectively the thickness of the outer and the inner face sheets and k = 1 when i = t, and k = 0 when i = b.

2.3. Constitutive equations

The constitutive relations for the core are equal to:

$$\sigma_{rrc} = E_c\varepsilon_{rrc}, \quad \tau_{xrc} = G_{xc}\gamma_{xrc}, \quad \tau_{\phi rc} = G_{\phi c}\gamma_{\phi rc} \tag{7}$$

The constitutive equations for each face sheet based on classical laminate theory are defined by the following stress resultant-displacement relations [17]:

$$\begin{aligned} N_{kl,i} &= A_{mn,i}\varepsilon_{kl0,i} + B_{mn,i}\kappa_{kl0,i} \\ M_{kl,i} &= B_{mn,i}\varepsilon_{kl0,i} + D_{mn,i}\varepsilon_{kl0,i} \quad (i=t,b) \end{aligned} \tag{8}$$

where N_{kl} and M_{kl} are stress and moment resultants, A_{mn}, B_{mm} and D_{mn} (m, n = 1, 2, 6) are rigidities and ε_{kl0} and κ_{kl0} are the mid-plane strains and the curvatures of the shell.

Kinetic energy of the sandwich shell depends on the velocity and the acceleration of its components. In general, the displacement distributions through the thickness of the core are very close to be linear, if the loads are smoothly distributed, and they are non-linear, if the loads are localized or concentrated. Since the vibration analysis of the cylindrical sandwich shell includes dynamic loads that are well distributed along the shell, the assumption of linearity in velocity and acceleration fields in the core are justified [13].

Note that these fields are used only to determine the kinetic energy. The accurate non-linear distributions that have been determined later from the solution of the equilibrium equations of the core are exploited as additional governing equations [see Eqs. (42), (44) and (45)].

The linear longitudinal, circumferential and radial displacements in the core can be expressed by:

$$\begin{aligned} u_c(r,\phi,x,t) &= [u_{ct}(\phi,x,t) - u_{cb}(\phi,x,t)]\frac{r - r_{bc}}{r_{tc} - r_{bc}} + u_{cb}(\phi,x,t) \\ v_c(r,\phi,x,t) &= [v_{ct}(\phi,x,t) - v_{cb}(\phi,x,t)]\frac{r - r_{bc}}{r_{tc} - r_{bc}} + v_{cb}(\phi,x,t) \\ w_c(r,\phi,x,t) &= [w_t(\phi,x,t) - w_b(\phi,x,t)]\frac{r - r_{bc}}{r_{tc} - r_{bc}} + w_b(\phi,x,t) \end{aligned} \tag{9}$$

where r_{tc} and r_{bc} are the radii of curvature at the outer and the inner core–face interfaces. Incorporation of the compatibility conditions (6) into Eq. (9), together with the rotations of the face sheets β_{xxi} and $\beta_{\phi\phi i}$ (i = t, b) as defined in Eq. (4), leads to the expression of the acceleration in the core as follows:

$$\begin{aligned} \ddot{u}_c &= \ddot{u}_{0t}\left(\frac{r - r_{bc}}{r_{tc} - r_{bc}}\right) + \ddot{u}_{0b}\left(1 - \frac{r - r_{bc}}{r_{tc} - r_{bc}}\right) \\ &\quad + \ddot{w}_{t,x}k_t r_t\left(\frac{r - r_{bc}}{r_{tc} - r_{bc}}\right) + \ddot{w}_{b,x}k_b r_b\left(\frac{r - r_{bc}}{r_{tc} - r_{bc}} - 1\right) \\ \ddot{v}_c &= \ddot{v}_{0t}(1 - k_t)\left(\frac{r - r_{bc}}{r_{tc} - r_{bc}}\right) + \ddot{v}_{0b}(1 + k_b)\left(1 - \frac{r - r_{bc}}{r_{tc} - r_{bc}}\right) \\ &\quad + \ddot{w}_{t,\phi}k_t\left(\frac{r - r_{bc}}{r_{tc} - r_{bc}}\right) - \ddot{w}_{b,\phi}k_b\left(1 - \frac{r - r_{bc}}{r_{tc} - r_{bc}}\right) \\ \ddot{w}_c &= \ddot{w}_t\left(\frac{r - r_{bc}}{r_{tc} - r_{bc}}\right) + \left(1 - \frac{r - r_{bc}}{r_{tc} - r_{bc}}\right)\ddot{w}_b \end{aligned} \tag{10a}$$

where

$$k_i = \frac{d_i}{2r_i} \quad (i=t,b) \tag{10b}$$

2.4. Equations of motion and boundary conditions

The derivation of the governing equations and the boundary conditions is based on the Hamilton's principle of the minimization of the Lagrangian $L$of the deformed system:

$$\delta L = \delta\int_{t_1}^{t_2}(T - U)dt = 0 \tag{11}$$

where U is the potential energy of the body (energy of the elastic deformation) and T is the kinetic energy. The first variation of the potential energy of the cylindrical shell is

$$\delta U = \int_{V_t} (\sigma_{xxt}\delta\varepsilon_{xxt} + \sigma_{\phi\phi t}\delta\varepsilon_{\phi\phi t} + \tau_{x\phi t}\delta\gamma_{x\phi t})dV_t + \int_{V_b} (\sigma_{xxb}\delta\varepsilon_{xxb} + \sigma_{\phi\phi b}\delta\varepsilon_{\phi\phi b} + \tau_{x\phi b}\delta\gamma_{x\phi b})dV_b + \int_{V_c} (\tau_{xrc}\delta\gamma_{xrc} + \tau_{\phi rc}\delta\gamma_{\phi rc} + \sigma_{rrc}\delta\varepsilon_{rrc})dV_c \quad (12)$$

where $\sigma_{xx\ i}$, $\sigma_{\phi\phi i}$, $\tau_{x\phi i}$, $\varepsilon_{xx\ i}$, $\varepsilon_{\phi\phi i}$, $\gamma_{x\phi i}$ ($i = t, b$) are the in-plane normal and shear stresses and strains in the face sheets; τ_{xrc}, $\tau_{\phi rc}$, σ_{rrc}, γ_{xrc}, $\gamma_{\phi rc}$, ε_{rrc}, the radial and shear stresses and strains in the core, and V_t, V_b, V_c the appropriate volumes of the outer and inner face sheets and the core, respectively.

The first variation of the kinetic energy can be obtained by assuming the homogeneous initial conditions and integration by parts with respect to the time coordinate:

$$\delta T = \int_{V_t} \rho_t(\ddot{u}_t\delta u_t + \ddot{v}_t\delta v_t + \ddot{w}_t\delta w_t)dV_t + \int_{V_b} \rho_b(\ddot{u}_b\delta u_b + \ddot{v}_b\delta v_b + \ddot{w}_b\delta w_b)dV_b + \int_{V_c} \rho_c(\ddot{u}_c\delta u_c + \ddot{v}_c\delta v_c + \ddot{w}_c\delta w_c)dV_c \quad (13)$$

where u_i, v_i, w_i ($i = t, b, c$) are longitudinal, circumferential and radial displacements and of the face sheets and the core.

Minimization of the Lagrangian L of the deformed system $\delta L = \delta(T - U) = 0$ is performed making use of the kinematic relations (1), the compatibility requirements (6), equations for the internal resultants (8) and the acceleration in the core (10). This provides nine equations of motion of the cylindrical sandwich shell as follows:

$$-r_t N^t_{x,x} - N^t_{x\phi,\phi} + r_{tc}\tau_{xrc}(r = r_{tc}) + r_t I_{0t}\ddot{u}_{0t} + K_1\ddot{u}_{0t} + K_2\ddot{u}_{0b} + K_1\frac{d_t}{2}\ddot{w}_{t,x} + K_2\frac{d_b}{2}\ddot{w}_{b,x} = 0 \quad (14)$$

$$-r_b N^b_{x,x} - N^b_{x\phi,\phi} - r_{bc}\tau_{xrc}(r = r_{bc}) + r_b I_{0b}\ddot{u}_{0b} + K_2\ddot{u}_{0t} + K_3\ddot{u}_{0b} + K_2\frac{d_t}{2}\ddot{w}_{t,x} + K_3\frac{d_b}{2}\ddot{w}_{b,x} = 0 \quad (15)$$

$$-N^t_{\phi,\phi} - \frac{M^t_{\phi,\phi}}{r_t} + r_{tc}(1-k_t)\tau_{\phi rc}(r = r_{tc}) + \left(I_{0t}r_t + \frac{I_{2t}}{r_t}\right)\ddot{v}_{0t} + (1-k_t)^2K_1\ddot{v}_{0t} + (1-k_t)(1+k_b)K_2\ddot{v}_{0b} + \left(k_t(1-k_t)K_1 - \frac{I_{2t}}{r_t}\right)\ddot{w}_{t,\phi} - k_b(1-k_t)K_2\ddot{w}_{b,\phi} - r_t N^t_{x\phi,x} - M^t_{x\phi,x} = 0 \quad (16)$$

$$-N^b_{\phi,\phi} - \frac{M^b_{\phi,\phi}}{r_t} - r_{bc}(1+k_b)\tau_{\phi rc}(r = r_{bc}) + \left(I_{0b}r_b + \frac{I_{2b}}{r_b}\right)\ddot{v}_{0b} + (1-k_t)(1+k_b)K_2\ddot{v}_{0t} + (1+k_b)^2K_3\ddot{v}_{0b} + k_t(1+k_b)K_2\ddot{w}_{t,\phi} - \left(k_b(1+k_b)K_3 + \frac{I_{2b}}{r_b}\right)\ddot{w}_{b,\phi} - r_b N^b_{x\phi,x} - M^b_{x\phi,x} = 0 \quad (17)$$

$$N^t_\phi - \frac{M^t_{\phi,\phi\phi}}{r_t} - r_t M^t_{xx,x} - 2M^t_{x\phi,x\phi} - r_{tc}k_t\tau_{\phi rc,\phi}(r = r_{tc}) + r_{tc}\sigma_{rrc}(r = r_{tc}) - r_{tc}\frac{d_t}{2}\tau_{xrc,x}(r = r_{tc}) + (r_t I_{0t} + K_1)\ddot{w}_t + K_2\ddot{w}_b - \left(k_t(1-k_t)K_1 - \frac{I_{2t}}{r_t}\right)\ddot{v}_{0t,\phi} - k_t(1+k_b)K_2\ddot{v}_{0b,\phi} - \left(k_t^2K_1 + \frac{I_{2t}}{r_t}\right)\ddot{w}_{t,\phi\phi} + k_t k_b K_2\ddot{w}_{b,\phi\phi} - K_1\frac{d_t}{2}\ddot{u}_{0t,x} - K_2\frac{d_t}{2}\ddot{u}_{0b,x} - K_1\left(\frac{d_t}{2}\right)^2\ddot{w}_{t,xx} - K_2\left(\frac{d_t d_b}{2}\right)\ddot{w}_{b,xx} + r_t I_{2t}\ddot{w}_{t,xx} = 0 \quad (18)$$

$$N^b_\phi - \frac{M^b_{\phi,\phi\phi}}{r_b} - r_b M^b_{xx,x} - 2M^b_{x\phi,x\phi} - r_{bc}k_b\tau_{\phi r,\phi}(r = r_{bc}) - r_{bc}\sigma_{rrc}(r = r_{bc}) - r_{bc}\frac{d_b}{2}\tau_{xrc,x}(r = r_{bc}) + K_2\ddot{w}_t + (r_b I_{0b} + K_3)\ddot{w}_b - k_b(1-k_t)K_2\ddot{v}_{0t,\phi} + \left(k_b(1+k_b)K_3 + \frac{I_{2b}}{r_b}\right)\ddot{v}_{0b,\phi} + k_t k_b K_2\ddot{w}_{t,\phi\phi} - \left(k_b^2K_3 + \frac{I_{2b}}{r_b}\right)\ddot{w}_{b,\phi\phi} - K_2\frac{d_b}{2}\ddot{u}_{0t,x} - K_3\frac{d_b}{2}\ddot{u}_{0b,x} - K_2\left(\frac{d_t d_b}{2}\right)\ddot{w}_{t,xx} - K_3\left(\frac{d_b}{2}\right)^2\ddot{w}_{b,xx} + r_b I_{2b}\ddot{w}_{b,xx} = 0 \quad (19)$$

$$\tau_{\phi rc} + (r\tau_{\phi rc})_{,r} = 0 \quad (20)$$

$$(r\tau_{xrc})_{,r} = 0 \quad (21)$$

$$\tau_{\phi rc,\phi} + r\tau_{xrc,x} + (r\sigma_{rrc})_{,r} = 0 \quad (22)$$

The following coefficients are introduced via the geometric characteristics of the shell (see Fig. 1) and the densities of the core and the face sheets.

$$I_{ji} = \int \rho_i z_i^j dz, \quad k_i = \frac{d_i}{2r_i} \quad (i = t, b)$$

$$K_1 = \frac{\rho_c t_c(3r_{tc} + r_{bc})}{12}, \quad K_2 = \frac{\rho_c t_c(r_{tc} + r_{bc})}{12}, \quad K_3 = \frac{\rho_c t_c(r_{tc} + 3r_{bc})}{12}$$

The boundary conditions at the edge of the shell for each face sheet and the core can be obtained as follows:

For the outer and the inner face sheets ($i = t, b$) at $\phi = \phi_0$ or ϕ_1:

$$u_{0i} = \tilde{u}_{0i} \quad \text{or} \quad N^i_{x\phi} = 0 \quad (23)$$

$$v_{0i} = \tilde{v}_{0i} \quad \text{or} \quad N^i_\phi + \frac{1}{r_i}M^i_\phi = 0 \quad (24)$$

$$w_t = \tilde{w}_t \quad \text{or} \quad \frac{1}{r_t}M^t_{\phi,\phi} - 2M^t_{x\phi,x} - \frac{I_{2t}}{r_t}\ddot{v}_{0t} - r_t I_{2t} w_{t,x} + r_t k_t \tau^t_{\phi rc} + k_t(1-k_t)K_1\ddot{v}_t + k_t(1+k_b)K_2\ddot{v}_b + \left(k_t^2K_1 - k_t k_b K_2\right)w_{t,\phi} = 0 \quad (25)$$

$$w_b = \tilde{w}_b \quad \text{or} \quad \frac{1}{r_t}M^b_{\phi,\phi} - 2M^b_{x\phi,x} - \frac{I_{2b}}{r_b}\ddot{v}_{0b} - r_b I_{2b} w_{b,x} + r_b k_b \tau^b_{\phi rc} - k_b(1-k_t)K_2\ddot{v}_t - k_b(1+k_b)K_3\ddot{v}_b + \left(k_b^2K_3 - k_t k_b K_2\right)w_{t,\phi} = 0 \quad (26)$$

$$w_{i,\phi} = \tilde{w}_{i,\phi} \quad \text{or} \quad M^i_\phi = 0 \quad (27)$$

$$w_c = \tilde{w}_c \quad \text{or} \quad \tau_{\phi rc} = 0 \quad (28)$$

For the outer and the inner face sheets, ($i = t, b$) at $y = 0$ or L:

$$u_{0i} = \tilde{u}_{0i} \quad \text{or} \quad r_i N_x^i = 0 \tag{29}$$

$$v_{0i} = \tilde{v}_{0i} \quad \text{or} \quad r_i N_{x\phi}^i + M_{x\phi}^i = 0 \tag{30}$$

$$w_i = \tilde{w}_i \quad \text{or} \quad r_i M_{x,x}^i + r_i \frac{d_i}{2} \tau_{xrc}^i = 0 \tag{31}$$

$$w_{i,x} = \tilde{w}_{i,x} \quad \text{or} \quad M_x^i = 0 \tag{32}$$

$$w_{i,\phi} = \tilde{w}_{i,\phi} \quad \text{or} \quad M_{x\phi}^i = 0 \tag{33}$$

$$w_c = \tilde{w}_c \quad \text{or} \quad \tau_{xrc} = 0 \tag{34}$$

2.5. Core stress and displacements fields

The first six governing Eqs. (14)–(19) are the equations of motion of the differential elements for outer and inner face sheets. The last three equations (20)–(22) describe the equilibrium of the elastic core medium in the cylindrical coordinates. They are uncoupled from the rest of the system and may be solved independently.

The core stress and the displacements field must first be determined in order to describe the equations of motion in terms of displacements and shear stresses. The stress and displacements fields within the core have been determined through the closed-form solution of the equilibrium equations [see Eqs. (20)–(22)] and the compatibility requirement of the displacements at the outer and the inner face–core interfaces [see Eq. (6)].

The relations for the shear stresses of the core are determined by solving Eqs. (20) and (21):

$$\tau_{\phi rc}(r, \phi, x) = \frac{\tau_\phi(\phi, x)}{r^2} \tag{35}$$

$$\tau_{xrc}(r, \phi, x) = \frac{\tau_x(\phi, x)}{r} \tag{36}$$

The normal stress within the core in the radial direction is determined using Eqs. (22), (35) and (36) and the compatibility requirement of the vertical displacements at the outer and the inner face–core interfaces, i.e. the last expression in Eq. (6), and it is:

$$\sigma_{rrc} = \frac{\tau_{\phi,\phi}}{r^2}\left(\frac{r(r_t - r_b)}{r_t r_b Ln(r_b/r_t)} + 1\right) - \left(\frac{(r_t - r_b)}{rLn(r_b/r_t)} + 1\right)\tau_{x,x} + \frac{E_c}{rLn(r_b/r_t)}(w_b - w_t) \tag{37}$$

with the help of kinematic relations, i.e. Eq. (3), the shear and radial stresses in the core, can be defined in the following constitutive relations:

$$\sigma_{rrc} = E_c w_{c,r} \tag{38}$$

$$\tau_{xrc} = G_{xc}(w_{c,x} + u_{c,r}) \tag{39}$$

$$\tau_{\phi rc} = G_{\phi c}\left(v_{c,r} + \frac{w_{c,\phi} - v_c}{r}\right) \tag{40}$$

Substituting the obtained radial stress in Eq. (37), into the first constitutive relation for the core, i.e. Eq. (38), and performing integration with respect to r, it is possible to obtain the radial displacement in the core as follows:

$$w_c = -\frac{\tau_{\phi,\phi}}{E_c r} - \frac{\tau_{x,x}}{E_c} r + \frac{c_1}{E_c} Lnr + c_2 \tag{41}$$

The radial displacement in the core has to satisfy the compatibility conditions in Eq. (6), at the outer and the inner interfaces, in the radial direction:

$$w_c = w_b + \frac{\tau_{\phi,\phi}}{E_c}\left[\frac{1}{r_{bc}} - \frac{1}{r} + \frac{k_0 Ln(r/r_{bc})}{Ln(r_{bc}/r_{tc})}\right] - \frac{\tau_{x,x}}{E_c}\left[r - r_{bc} + \frac{(r_{bc} - r_{tc})Ln(r/r_{bc})}{Ln(r_{bc}/r_{tc})}\right] + (w_b - w_t)\frac{Ln(r/r_{bc})}{Ln(r_{bc}/r_{tc})} \tag{42}$$

where

$$k_0 = \frac{r_{tc} - r_{bc}}{r_{tc} r_{bc}}$$

To obtain circumferential displacements in the core, one has to solve the third constitutive relation in Eq. (40), which is an ordinary differential equation for the core and by taking into account the Eq. (35):

$$r v_{c,r} - v_c = \frac{\tau_\phi}{rG_{c\phi}} - w_{c,\phi} \tag{43}$$

Furthermore, it is necessary to fulfill the compatibility conditions in Eq. (6), and use the relations for the face rotations, i.e. Eqs. (1) and (2) simultaneously:

$$\begin{aligned} v_c = {} & \frac{r}{r_{bc}}(1 + k_b) v_{ob} + \left[1 - \frac{r}{r_{bc}}(1 + k_b)\right] w_{b,\phi} + \frac{1}{2}\left(\frac{r^2 - r_{bc}^2}{r_{bc}^2 r}\right)\frac{\tau_\phi}{G_{c\phi}} \\ & + \frac{\tau_{\phi,\phi\phi}}{E_c}\left[-\frac{(r - r_{bc})^2}{2rr_{bc}^2} + \frac{k_0(1 - r/r_{bc} + Ln(r/r_{bc}))}{Ln(r_{bc}/r_{tc})}\right] \\ & - (w_{t,\phi} - w_{b,\phi})\frac{(1 - r/r_{bc} + Ln(r/r_{bc}))}{Ln(r_{bc}/r_{tc})} \\ & + \frac{\tau_{x,x\phi}}{E_c}\left[\frac{(r_{bc} - r_{tc})(1 + Ln(r/r_{bc}) - (r/r_{bc}))}{Ln(r_{bc}/r_{tc})} + r_{bc} - r + rLn(r/r_{bc})\right] \end{aligned} \tag{44}$$

Also to obtain longitudinal displacements in the core, u, it is necessary to integrate the second constitutive relation for the core, i.e. Eq. (39) with respect to r, and by taking into account the Eq. (36) and fulfill the compatibility conditions in Eq. (6):

$$\begin{aligned} u_c = {} & u_{0b} + \left(r_{bc} - r - \frac{d_b}{2}\right) w_{b,x} + \frac{\tau_x}{G_{cx}} Ln\left(\frac{r}{r_{bc}}\right) \\ & - \frac{\tau_{\phi,x\phi}}{E_c}\left[\frac{r}{r_{bc}} - 1 + Ln\left(\frac{r_{bc}}{r}\right) + \frac{k_0(r_{bc} + r(Ln(r/r_{bc}) - 1))}{Ln(r_{bc}/r_{tc})}\right] \\ & + \frac{\tau_{x,xx}}{E_c}\left[\frac{r^2 + r_{bc}^2}{2} - rr_{bc} - \frac{(r_{bc} - r_{tc})(r_{bc} + r(Ln(r/r_{bc}) - 1))}{Ln(r_{bc}/r_{tc})}\right] \\ & - (w_{b,x} - w_{t,x})\frac{r_{bc} + r(Ln(r/r_{bc}) - 1)}{Ln(r_{bc}/r_{tc})} \end{aligned} \tag{45}$$

It can be noted that, the displacement distributions in the longitudinal, circumferential and the radial directions in Eqs. (42), (44) and (45) are in general non-linear.

The use of compatibility conditions in Eq. (6), at the inner interface together with the expressions for u_c and v_c, gives the required seventh and eighth equations along with the other six Eqs. (14)–(19):

$$\begin{aligned} & \frac{r_t}{r_b}(1 + k_b) v_{ob} + (k_t - 1) v_{0t} + \left[1 - \frac{r_t}{r_b}(1 + k_b)\right] w_{b,\phi} - k_t w_{t,\phi} + \frac{1}{2}\left(\frac{r_t^2 - r_b^2}{r_b^2 r_t}\right)\frac{\tau_\phi}{Gc_\phi} \\ & + \frac{\tau_{\phi,\phi\phi}}{E_c}\left[-\frac{(r_t - r_b)^2}{2r_t r_b^2} + \frac{k_0(1 - r_t/r_b + Ln(r_t/r_b))}{Ln(r_b/r_t)}\right] \\ & - (w_{t,\phi} - w_{b,\phi})\frac{(1 - r_t/r_b + Ln(r_t/r_b))}{Ln(r_b/r_t)} \\ & + \frac{\tau_{x,x\phi}}{E_c}\left[\frac{(r_b - r_t)(1 + Ln(r_t/r_b) - (r_t/r_b))}{Ln(r_b/r_t)} + r_b - r_t + r_t Ln(r_t/r_b)\right] = 0 \end{aligned} \tag{46}$$

$$u_{0b} - u_{0t} - \frac{d_t}{2} w_{t,x} + \left(r_b - r_t - \frac{d_b}{2}\right) w_{b,x} - (w_{b,x} - w_{t,x}) \frac{r_b + r_t(Ln(r_t/r_b) - 1)}{Ln(r_b/r_t)} + \frac{\tau_x}{G_{cx}} Ln\left(\frac{r_t}{r_b}\right) - \frac{\tau_{\phi,x\phi}}{E_c}\left[\frac{r_t}{r_b} - 1 + Ln\left(\frac{r_b}{r_t}\right) + \frac{k_0(r_b + r_t(Ln(r_t/r_b) - 1))}{Ln(r_b/r_t)}\right] + \frac{\tau_{x,xx}}{E_c}\left[\frac{r_t^2 + r_b^2}{2} - r_t r_b - \frac{(r_b - r_t)(r_b + r_t(Ln(r_t/r_b) - 1))}{Ln(r_b/r_t)}\right] = 0 \tag{47}$$

The governing equations of motion are formulated in terms of the following eight unknowns: the circumferential and longitudinal displacements of the mid-plane of the outer and the inner face sheets, the vertical deflections of the outer and the inner face sheets and the two vertical shear stresses in the core. The first six equations are determined through substitution of the constitutive relations in Eq. (8), in the equations of motion of the face sheets, i.e. Eqs. (14)–(19) with the help of Eqs. (38)–(40). Also, the additional two necessary equations have been derived using the circumferential and longitudinal displacement distributions in Eqs. (46) and (47). Finally, the governing equations of the composite sandwich cylindrical shell may be expressed in the compact matrix form as follows:

$$\widetilde{M}\ddot{X} + \widetilde{K}X = 0 \tag{48}$$

where $\widetilde{M}$ is the mass matrix, $\widetilde{K}$ the stiffness matrix and X the vector of unknown variables defines as follows:

$$X^T = \{u_{0t} u_{0b} v_{0t} v_{0b} w_t w_b \tau_\phi \tau_x\} \tag{49}$$

3. Free vibration of composite sandwich cylindrical shell

To investigate the free vibrations of the sandwich shell, the boundary conditions are considered to be simply-supported on the edges of the outer and the inner face sheets. The face sheets consist of a specially orthotropic fibre laminate composite, with unsymmetric lay-up, where $A_{16} = A_{26} = D_{16} = D_{26} = B_{16} = B_{26} = 0$. The solution in such case is analytical and their displacements are in the following form [17]:

$$X = \begin{Bmatrix} u_{0t}(\phi, x) \\ u_{0b}(\phi, x) \\ v_{0t}(\phi, x) \\ v_{0b}(\phi, x) \\ w_t(\phi, x) \\ w_b(\phi, x) \\ \tau_\phi(\phi, x) \\ \tau_x(\phi, x) \end{Bmatrix} e^{j\omega t} = \begin{Bmatrix} C_{ut}\sin(n\pi\phi/\alpha_0)\cos(m\pi x/L) \\ C_{ub}\sin(n\pi\phi/\alpha_0)\cos(m\pi x/L) \\ C_{vt}\cos(n\pi\phi/\alpha_0)\sin(m\pi x/L) \\ C_{vb}\cos(n\pi\phi/\alpha_0)\sin(m\pi x/L) \\ C_{wt}\sin(n\pi\phi/\alpha_0)\sin(m\pi x/L) \\ C_{wb}\sin(n\pi\phi/\alpha_0)\sin(m\pi x/L) \\ C_{\tau\phi}\cos(n\pi\phi/\alpha_0)\sin(m\pi x/L) \\ C_{\tau x}\sin(n\pi\phi/\alpha_0)\cos(m\pi x/L) \end{Bmatrix} e^{j\omega t} \tag{50}$$

C_{ut}, C_{ub}, C_{vt}, C_{vb}, C_{wt}, C_{wb}, $C_{\tau\phi}$, $C_{\tau x}$ are the amplitudes of the vibration; j the complex notation; ω the natural frequency of vibration measured in [rad/s]; m and n the orders of the natural vibration mode.

The solution is determined through substitution of Eq. (50) into the governing Eq. (48), which yields a set of homogeneous algebraic equations, instead of the set of partial differential equations. Thus, the solution of the partial differential equations is replaced

Table 1
Materials properties used for the analysis.

Material properties	Face sheets	Core
(0/90/0/core/0/90/0) Ref. [18]	$E_1 = 24.51$ GPa, $E_2 = E_3 = 7.77$ GPa $G_{12} = G_{13} = 3.34$ GPa, $G_{23} = 1.34$ GPa, $\nu_{12} = \nu_{13} = 0.078$, $\nu_{23} = 0.49$, $\rho = 1800$ kg/m^3	$E_1 = E_2 = E_3 = 0.10363$ GPa, $G_{12} = G_{13} = G_{23} = 0.05$ GPa, $\nu = 0.33$, $\rho = 130$ kg/m^3
(0/90/core/0/90) Ref. [4]*	$E_1 = 131$ GPa, $E_2 = E_3 = 10.34$ GPa $G_{12} = G_{13} = 6.895$ GPa, $G_{13} = 6.205$ GPa $\nu_{12} = \nu_{13} = 0.22$, $\nu_{23} = 0.49$, $\rho = 1627$ kg/m^3	$E_1 = E_2 = E_3 = 0.00689$ GPa $G_{12} = G_{13} = G_{23} = 3.45$ GPa $\nu = 0$, $\rho = 94.195$ kg/m^3

Table 2
Dimensionless natural frequencies for the sandwich panel (0/90/0/core/0/90/0).

Modes (m, n)	Present	ANSYS	Discrepancy (%)	Analytical-HSDT (ESL) [18]	Discrepancy (%)	FEM–HSDT (ESL) [19]	Discrepancy (%)
(1, 1)	14.27	14.74	3.19	15.28	6.61	15.34	6.97
(1, 2)	26.31	26.83	1.94	28.69	8.29	30.18	12.82
(2, 1)	27.04	27.53	1.78	30.01	9.90	31.96	15.39
(2, 2)	34.95	35.60	1.82	38.86	10.01	40.94	14.63

Table 3
Dimensionless natural frequencies for sandwich panel (0/90/core/0/90).

h/a	Modes	Present model	ANSYS	LW [23]	ESL [22]
0.1	1,1	1.7586	1.6556	1.8480	4.8594
	1,2	2.9593	2.8247	3.2196	8.0187
	1,3	4.8312	4.6981	5.2234	11.7381
	2,2	4.1045	3.9641	4.2894	10.2966
	2,3	5.7681	5.6254	6.0942	13.4706
	3,3	7.4187	7.2783	7.6762	16.1320
0.01	1,1	12.0265	10.8913	11.9401	15.5093
	1,2	22.9918	22.7686	23.4017	39.0293
	1,3	35.5133	35.6493	36.1434	72.7572
	2,2	29.9178	28.2381	30.9432	54.7618
	2,3	40.9068	39.3818	41.4475	83.4412
	3,3	49.3416	45.4457	49.7622	105.3781

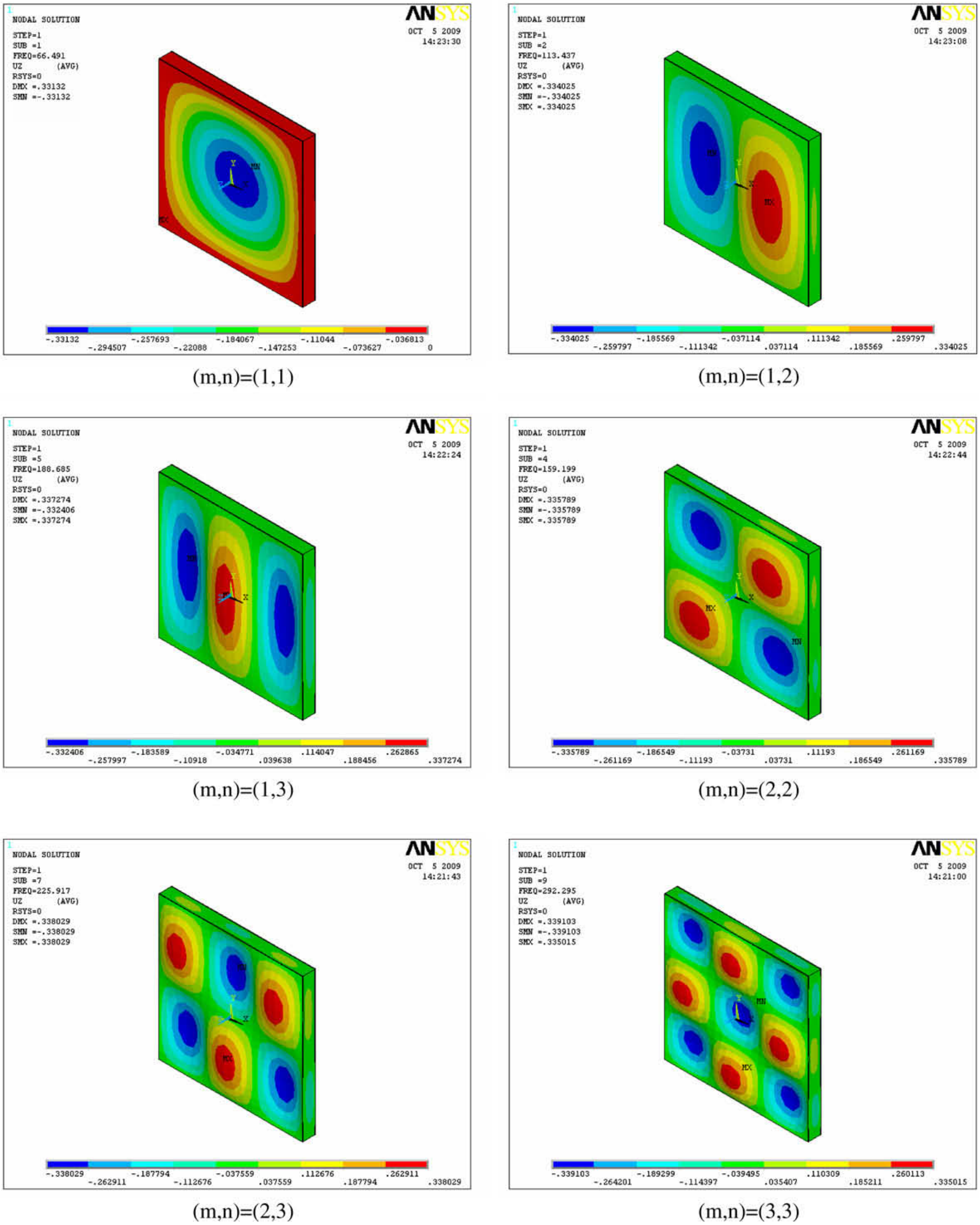

Fig. 2. Mode shapes of laminated sandwich panel (0/90/core/0/90).

Table 4
Dimensionless natural frequencies for antisymmetric sandwich cylindrical shells (0/90/core/0/90).

R/a	h/a	Present model	ANSYS	ESL–HSDT [4]	ESL–FSDT [4]
1	0.01	63.26908	64.62096	64.63986	64.80146
	0.1	5.65219	6.45819	7.71269	14.16395
2	0.01	33.86598	34.50038	35.90110	36.21419
	0.1	2.96392	3.70794	5.82454	14.02597
3	0.01	24.16771	24.80504	26.69465	27.11983
	0.1	2.19491	2.83296b	5.36505	14.00424

by an eigenvalue problem, with a mass and a stiffness matrix, where the square of the eigenfrequency equals to the eigenvalue and the constants are the corresponding eigenvectors:

$$(-\omega^2 M + K)C = 0 \tag{51}$$

where

$$C^T = \{C_{ut}\ C_{ub}\ C_{vt}\ C_{vb}\ C_{wt}\ C_{wb}\ C_{\tau\phi}\ C_{\tau x}\} \tag{52}$$

4. Results and discussion

4.1. Validation

In order to validate the present analytical method, the results obtained from the present solution are compared with the results reported in the literatures by other authors [18,4].

The first example deals with a simply supported sandwich panel having FRP face sheets and PVC foam core. The face sheets are made of glass polyester resins and the core is made of HEREX C70.130 PVC foam and their materials properties are presented in the first row of Table 1 [18]. The sandwich panel was made by (0/90/0/core/0/90/0) lay-ups.

In order to apply the developed model to a sandwich panel, the radii of the curvature in the present governing equations are set to be large in comparison with all other geometrical parameters of the shell. The first four dimensionless natural frequencies $\bar{\omega} = \omega a^2(\rho/E)_c^{1/2}/h$ [18] for a square sandwich panel with $h/a = 0.10$ and $h_c/h = 0.88$ are obtained by the present theory and compared with those obtained by reference [18] in Table 2. The analytical method used by Ref. [18] is obtained by higher order shear deformation theory [HSDT] considered equivalent single layer assumption.

As observed from Table 2, the eigenfrequency of the present model is lower than those values obtained by Ref. [18] and the maximum discrepancy is 10.01% which occurred at 4th wavenumber. The results show that the discrepancy between two theories increases at higher modes. In the same table, the results obtained by finite element method used Reddy's higher-order theory [19] are shown and compared with the present model results. Maximum discrepancy in this comparison is 14.63%, which is due to consideration of additional degrees of freedom for flexibility of the core in thickness direction, in the present model.

For further validation, the free vibration analysis of sandwich panel with a flexible core is carried out using parametric design language (APDL) of ANSYS commercial software. The element type used to model the composite face sheets of the sandwich panel is the eight-node layered structural shell 99. The flexible PVC core is modeled using the higher order solid 95 element. This element is defined by 20 nodes and can tolerate irregular shapes with compatible displacement shapes [20].

In the modeling of sandwich structures, the motion of face sheets modeled as laminates with nodes at their mid-planes is assumed to be different from that of the top surface of the core. In other words, the nodes of the shell elements which modeled the top and bottom face sheets have different degrees of freedom from that of the nodes defining the top and bottom surfaces of the core. This is because by simple coupling of the face sheet nodes with those of the core, a correct finite element model is not obtained. This problem is removed through the use of a set of user-defined constraint equations, which provides a more general means of relating the degrees of freedom values when compared to possible consideration of simple coupling. These constraint equations satisfied the continuity between the bottom of the top face sheet and the top surface of the core and similarly between the top of the bottom face sheet and the bottom surface of the core, i.e. Eq. (6), [21].

As can be seen from Table 2, the results of present theory are in good agreement with the results of present ANSYS model. Both of these models which applied the flexibility of the core, lead to lower eigenfrequencies compared to those of Refs. [18,19], in which the whole of sandwich panel is considered as a single layer.

In the second example, a five-layer (0/90/core/0/90), thin as well as thick sandwich panel has been analyzed. The physical properties used for this example are given in the 2nd row of Table 1. The ratio of thickness of core to face sheet (h_c/h_f) is considered to be 10. Results obtained from the present theories for thin panel ($h/a = 0.01$) and moderately thick panel ($h/a = 0.1$) are shown in Table 3 together with the results obtained by ESL theory, Ref. [22] and layerwise (LW) theory, Ref. [23]. Furthermore, the natu-

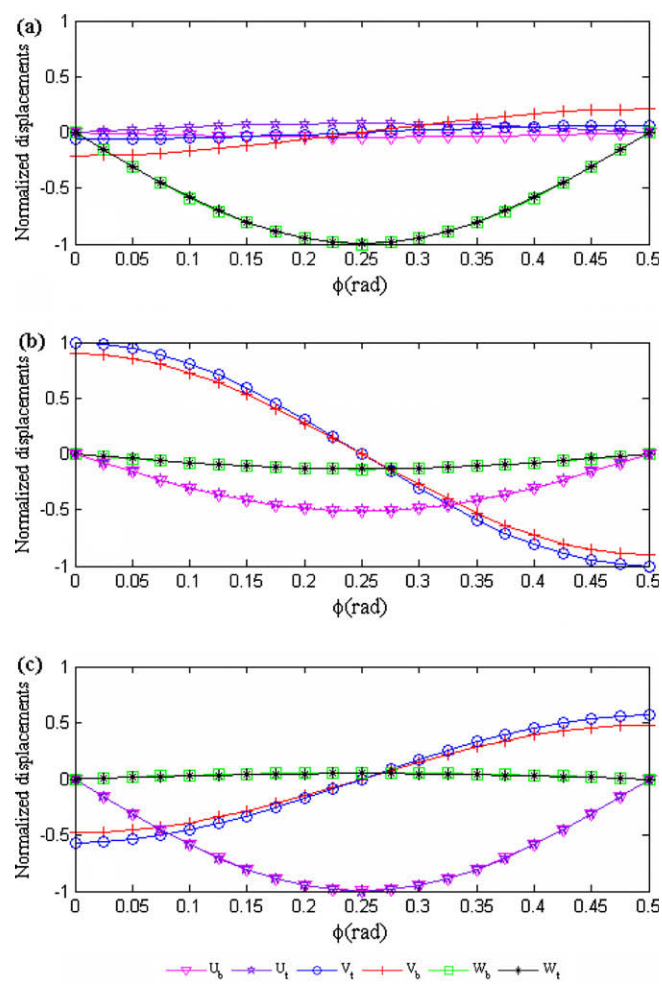

Fig. 3. Mode shapes of laminated sandwich cylindrical shell ($m = n = 1$) for various ϕ (antisymmetric).

Table 5
Natural frequencies of laminated sandwich cylindrical shells (0/90/0/core/0/90/0) $h/a = 0.10$, $h_c/h = 0.88$, $L = 1$ m ($m = n = 1$).

Frequencies (Hz)	$R = 1$ m, $a = 1$ m	$R = 2$ m, $a = 1$ m	Mode shape
ω_1	234.77	211.92	Anti-symmetric
ω_2	1186.8	1162.5	Anti-symmetric
ω_3	1491.5	1488.1	Anti-symmetric
ω_4	1958.1	1986.6	Symmetric
ω_5	2202.6	2180.1	Symmetric
ω_6	2321.2	2282.1	Symmetric

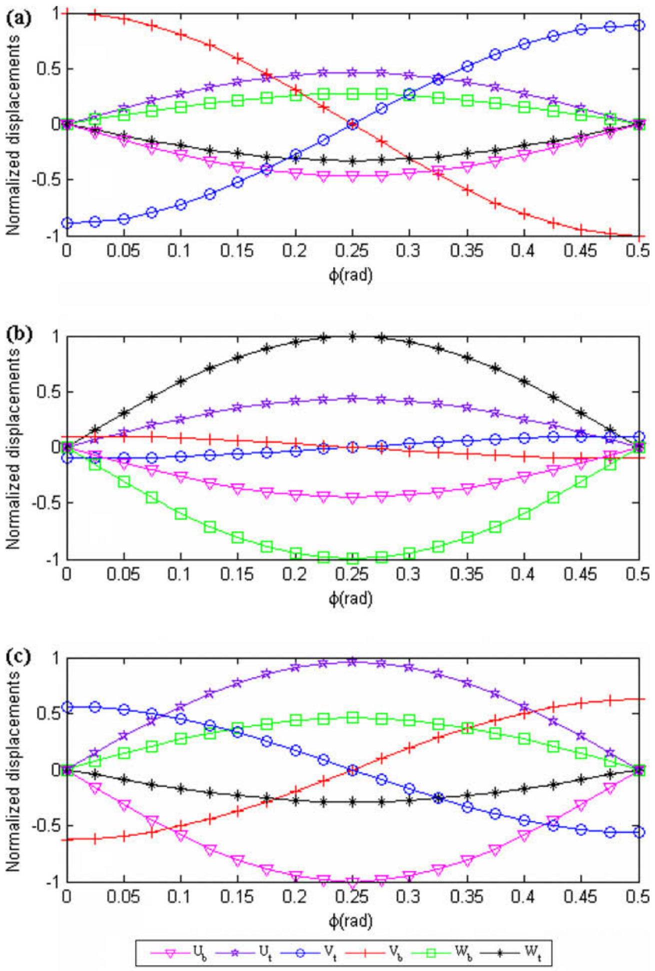

Fig. 4. Mode shapes of laminated sandwich cylindrical shell ($m = n = 1$) for various ϕ (symmetric).

ral frequencies acquired from ANSYS model as mentioned above, in addition to corresponding mode shapes are presented in Table 3 and Fig. 2 respectively. Natural frequencies are normalized by using relation $\bar{\omega} = \omega a^2(\rho/E_2)_f^{1/2}/h$. It can be observed from Table 3 that the results obtained by ESL theory obviously overestimate the natural frequencies in comparison with the proposed model and also LW theory. This can be attributed to the large differences in the stiffness between the face sheets and the core material. As a result, the ESL models overestimate the stiffness of the plate, while obtaining the equivalent material properties. However, the discrepancies are less for thin panels as compared to the thick panels. The advantage of the proposed theory can be observed obviously from the results of thick sandwich panel ($h/a = 0.1$). Thus, the present model can be used for thin as well as thick sandwich panels. Moreover, the results obtained by the proposed model are in close agreement with FEM results obtained by present ANSYS model as well as the results obtained by LW models [23].

In the next example, the variation of dimensionless fundamental frequency $\bar{\omega} = \omega a^2(\rho/E_2)_f^{1/2}/h$ with respect to face sheets [4] for five layer cylindrical sandwich shells having square planform is investigated. The face sheets are antisymmetric cross-ply and the values of dimensionless frequencies are obtained for different values of radius to width ratios in conjunction with thickness to width ratios. The values obtained by present model are compared with the results obtained analytically by Ref. [4] which used HSDT and first order shear deformation theory (FSDT) by considering equivalent single layer. The results are shown in Table 4. Furthermore, the FEM results obtained by present ANSYS model are included in this table. The core to face thickness ratio (h_c/h_f) is taken to be 10. The physical properties which are same as previous example, are used for antisymmetric (0/90/core/0/90) sandwich cylindrical shells [4].

Results indicate that the use of present higher order sandwich shell theory leads to lower eigenfrequencies compared to those of ordinary theories in Ref. [4]; for thin shells ($h/a = 0.01$), the results of various theories are in good agreement, but increasing thickness-to-side ratio ($h/a = 0.1$) leads to discrepancy and this is

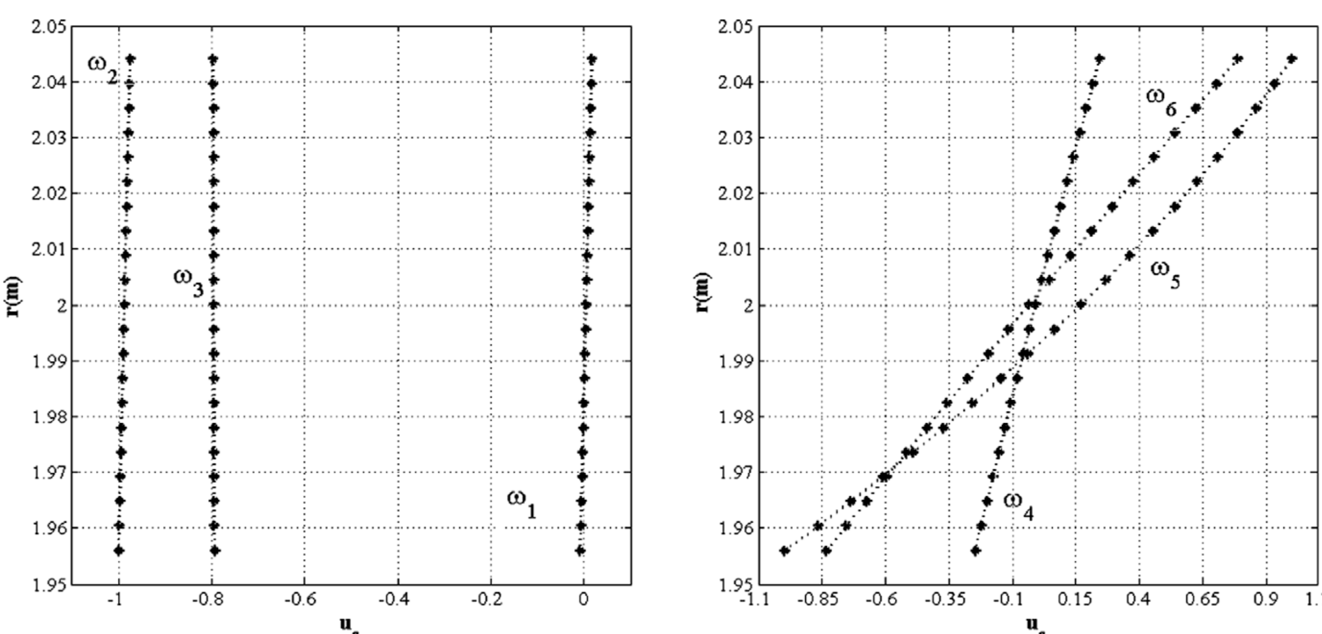

Fig. 5a. Distribution of the core longitudinal displacements u_c through its thickness.

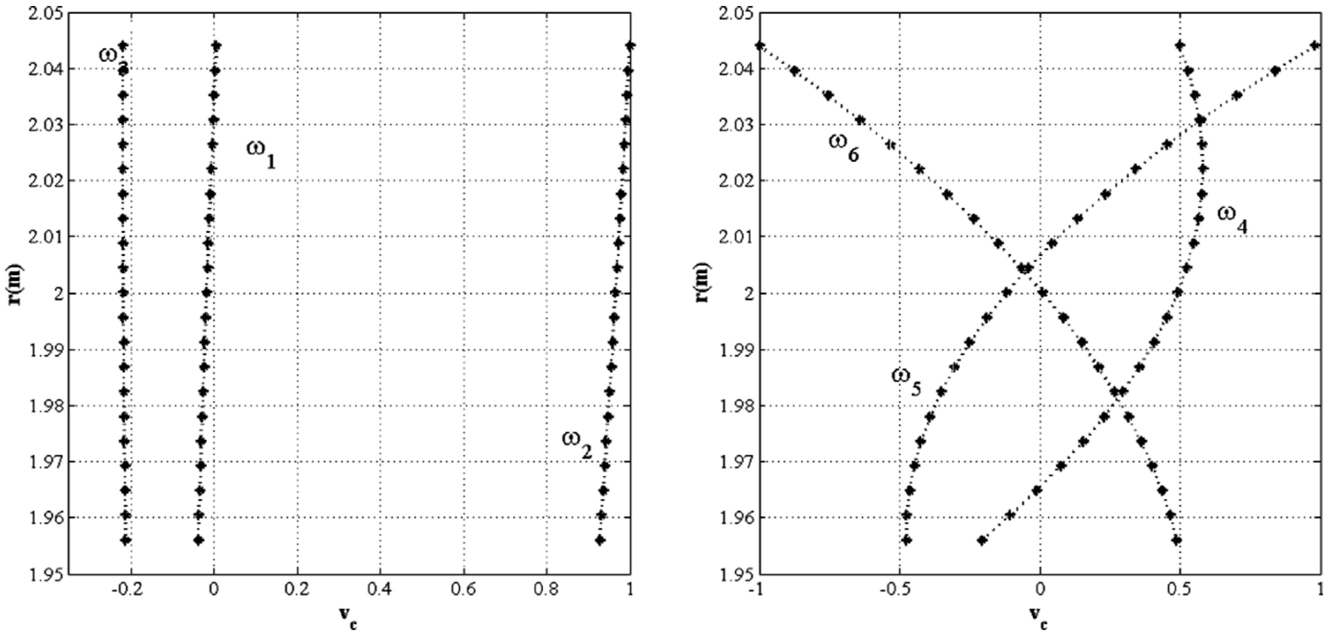

Fig. 5b. Distribution of the core circumferential displacements v_c through its thickness.

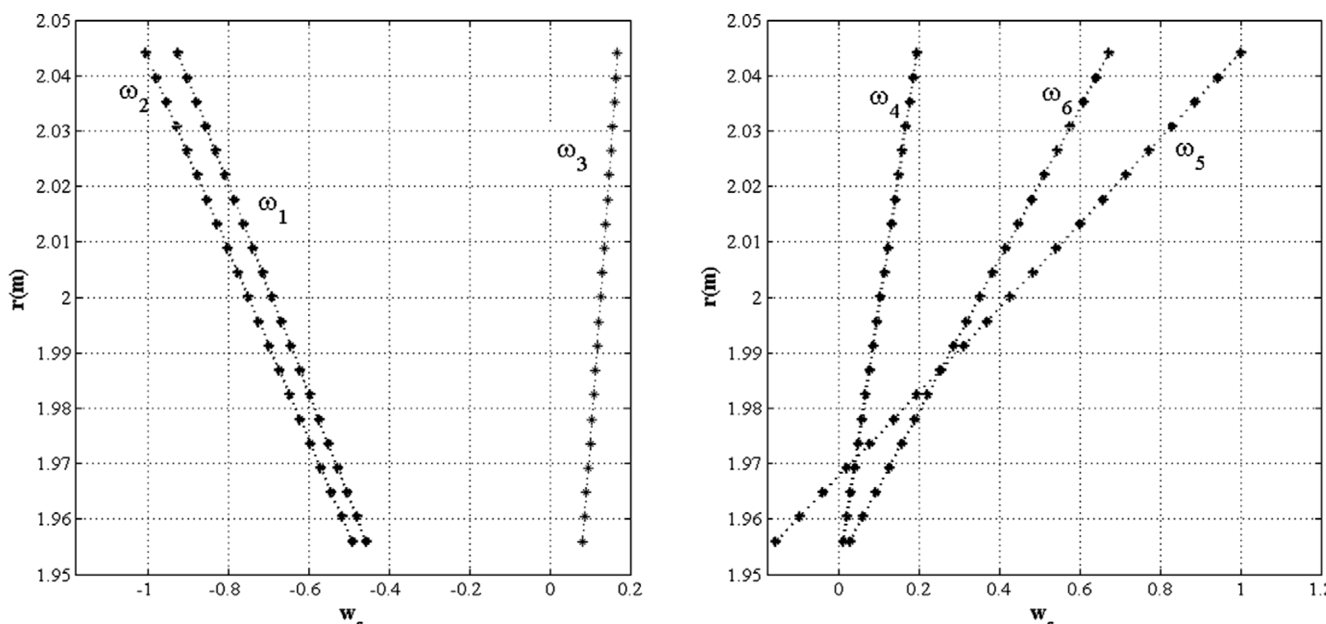

Fig. 5c. Distribution of the core radial displacements w_c through its thickness.

as the result of considering flexibility of the core by applying the additional degrees of freedom in the present model. Ordinary theories over-predict the natural frequency by a significant magnitude and the discrepancy drastically increases as the thickness of sandwich shell increases.

In other words, Ref. [4] in which the whole sandwich panel is assumed as a single layer, fails to consider the transverse flexibility of the core and therefore obtains higher values for the natural frequencies, whereas the present model yields more accurate results by considering the transverse compressibility of the core. In fact, by using ESL model in Ref. [4], the changes in the height of the core

Table 6
Natural frequencies of laminated sandwich cylindrical shell (0/90/0/core/0/90/0) with respect to the radius of the shell h/a = 0.10, h_c/h = 0.88, a = 1 m, L = 1 m (m = n = 1).

Frequencies	R = 1 m	R = 2 m	R = 3 m	R = 5 m	R = 10 m
ω_1	234.77	211.92	207.19	204.69	203.62
ω_2	1186.8	1162.5	1157.9	1155.6	1154.6
ω_3	1491.5	1488.1	1487.4	1487.1	1486.9
ω_4	1958.1	1986.6	1993.7	1997.7	1999.5
ω_5	2202.6	2180.1	2173.0	2046.9	2036.2
ω_6	2321.2	2282.1	2275.8	2272.8	2271.5

Table 7
Natural frequencies of laminated sandwich cylindrical shell (0/90/0/core/0/90/0) with respect to the length of the shell $h/a = 0.10$, $h_c/h = 0.88$, $a = 1$ m, $R = 2$ m ($m = n = 1$).

Frequencies	$L = 1$ m	$L = 2$ m	$L = 3$ m	$L = 5$ m	$L = 10$ m
ω_1	211.92	141.35	128.18	122.33	120.27
ω_2	1162.5	827.43	691.13	605.66	565.05
ω_3	1488.1	1168.6	1136.6	1121.9	1116.0
ω_4	1986.6	1775.0	1698.8	1656.2	1637.6
ω_5	2180.1	1989.9	1965.9	1955.1	1950.8
ω_6	2282.1	2195.5	2193.8	2193.1	2192.8

(compressibility) during the deformation of the sandwich panel is neglected, which results in inaccuracy, during analyzing the sandwich panels with flexible and thick cores. By increasing the (h/a) ratio, the effect of core flexibility increases and hence leads a higher discrepancies in the results.

Furthermore, the present formulation permits the existence of the mode shapes consisting of a relative displacement between the two face sheets. Such modes can not be determined by ordinary theories defined earlier, in which the shell is modeled as an equivalent single layer with higher order effects. This phenomenon is discussed in details in the next section.

4.2. Natural modes of cylindrical composite sandwich shell

By solving Eq. (51), each wavenumber produces six eigenvalues that are the eigenfrequencies of the sandwich shell with flexible core. These frequencies for composite sandwich cylindrical shell with the properties given in Ref. [18], for the first wavenumber $m = n = 1$ are presented in Table 5.

As mentioned in the previous section, the proposed computational model is also able to detect higher eigenfrequencies, which the various shell theories alongwith the high-order effects can not detect these values. In Table 5, the anti-symmetric modes involve a displacement pattern that is anti-symmetric with respect to the shell mid-plane, that is, the face sheets move in phase with each other. In contrast, in the symmetric modes, the displacements are symmetric with respect to the shell mid-plane, that is, the face sheets move out of phase with respect to each other. The six eigenmodes in form of normalized displacement, correspond to the second column of Table 5 are presented in Figs. 3 and 4.

It can be observed from these figures, that the vibration patterns of the sandwich cylindrical shell with flexible core are more complex than those of a homogeneous shell [17]. This is a direct consequence of low Young and shear modulus of the soft core.

In Fig. 3 the anti-symmetric modes, which correspond to the first three eigenmodes (ω_1–ω_3) are illustrated, while in Fig. 4, the symmetric modes, which correspond to the next three eigenmodes (ω_4–ω_6) are shown. However the natural modes of the cylindrical shell are different from those of the plate [16], since the natural modes of the cylindrical shell are of a mixed nature. For example, dominating the overall anti-symmetric bending of the entire shell at the first mode, Fig. 3a, is additionally accompanied by an insignificant local in-plane movement of the faces. Moreover in Fig. 3b and c, small transverse vibration of the core accompanies the second and third shear modes. The in-plane displacements of the various face sheets in Fig. 4, reveal that the outer and the inner face sheets are distorted perpendicular to each other. Also, out of plane mode (pumping mode [16]), which appears in Fig. 4b, happens together with the minor in-plane shear vibration. In this mode, the outer and the inner face sheets move opposite to each other in local bending.

The distributions of the displacements of the core, at $\phi = \alpha_0/4$ and $x = L/4$ and through its thickness, for the six eigenmodes, in longitudinal, circumferential and radial directions, appear in Fig. 5a–c respectively. The results reveal that the distribution of the displacements is linear for the first three modes which are anti-symmetric. The results of the second three modes, which are symmetric ones, are slightly non-linear for the longitudinal and radial displacements and have higher order non-linearity for the circumferential displacements. The linear distributions of displacements are in agreement with the assumed linear distributions of the accelerations of the core through its thickness [see Eq. (10)].

The influence of geometry parameters of sandwich cylindrical shell on its vibration behavior is also investigated. In Table 6, the effect of changing radius of the cylindrical shell on its frequencies and in Table 7, the results of changing length of the shell on the natural frequencies are shown. Increasing the radius and the length of the shell, leads to decreasing the natural frequency of the shell and the length has predominant effect. Also, the variation of six eigenfrequencies with respect to the sector angle of the shell

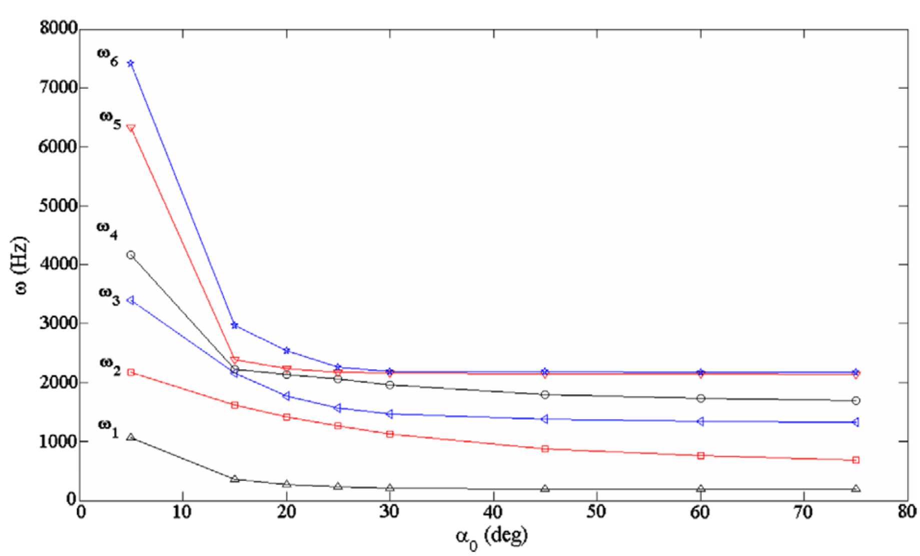

Fig. 6. Variation of six eigenfrequencies with respect to the sector angle of the shell.

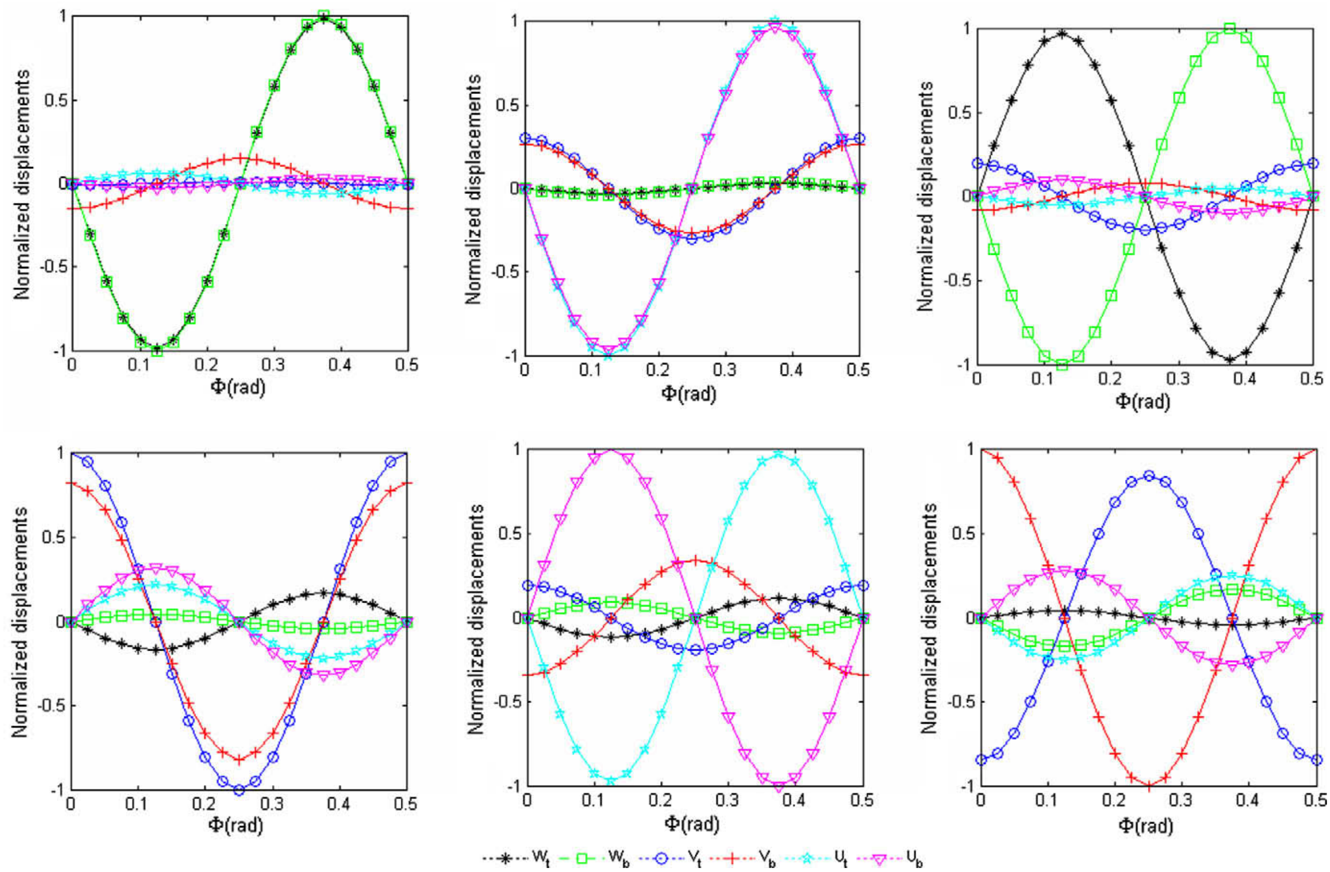

Fig. 7. Mode shapes of the laminated sandwich cylindrical shell ($m = 1$, $n = 2$).

is presented in Fig. 6. All values of eigenfrequencies are reduced at much higher rate for small sector angle, but it reaches to a threshold value at α_0 around 30°.

The present model is able to predict the relative displacements between the face sheets at higher mode numbers. For example, the results of second wavenumber ($m = 1$, $n = 2$) at $x = L/4$ are presented in Fig. 7.

5. Conclusions

This study considered the free vibration analysis of composite sandwich cylindrical shell with flexible core by the use of high-order sandwich panel theory (HSAPT). The HSAPT for beams and plates for the first time, is extended to cylindrical shells using the classical shell theory for the face sheets and the 3D elasticity solution for the core, and showed excellent predictions of behavior in free vibration analysis. The mathematical formulation uses the Hamilton's principle to derive the equations of motion alongwith the appropriate boundary conditions. By considering the flexibility of the sandwich core in the analysis, the model can achieve to the modes consisting of a relative displacement between the two face sheets which are not detected by other models. The results revealed that the sandwich shells with flexible core exhibit a complex behavior, and that the vibration patterns of the sandwich cylindrical shells are more complex than those of the homogeneous shells. Furthermore, it was observed that the natural modes of the sandwich shell are different from those of the sandwich plate and have a mixed mode nature.

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